

PRE Calc

Review for Final Exam *key*

Name _____ Period _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find an explicit rule for the n th term of the arithmetic sequence.

1) $-9, -5, -1, 3, \dots$ $a_1 = -9, d = 4$

1) _____

$$\boxed{a_n = -9 + 4(n-1)} \text{ or } \boxed{a_n = -9 + 4n - 4 = 4n - 13}$$

Find an explicit rule for the n th term of the sequence.

2) $4, -8, 16, -32, \dots$ $a_1 = 4, r = -2$

2) _____

$$\boxed{a_n = (4)(-2)^{n-1}}$$

Write the series using summation notation.

3) $-1 + 3 + 7 + 11 + \dots + 59$ Find n
 $a_1 = -1, d = 4, a_n = 59$

3) _____

$$\begin{aligned} 59 &= -1 + 4(n-1) \\ 60 &= 4n - 4 \quad n = 16 \\ 64 &= 4n \end{aligned}$$

$$\boxed{\sum_{n=1}^{16} 4n - 5}$$

$$\text{or } \boxed{\sum_{n=0}^{15} 4n - 1}$$

Write out the first five terms of the sequence.

4) $c_n = \frac{n+2}{3}$

4) _____

$$c_1 = \frac{1+2}{3} = 1$$

$$c_3 = \frac{3+2}{3} = \frac{5}{3}$$

$$c_5 = \frac{5+2}{3} = \frac{7}{3}$$

$$c_2 = \frac{2+2}{3} = \frac{4}{3}$$

$$c_4 = \frac{4+2}{3} = 2$$

$$\boxed{\left\{1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}\right\}}$$

Solve.

5) An auditorium has 25 rows with 10 seats in the first row, 12 in the second row, 14 in the third row, and so forth. How many seats are in the auditorium?

5) _____

$$n = 25, d = 2, a_1 = 10$$

$$\begin{aligned} a_{25} &= 10 + 2(25-1) \\ &= 10 + 48 \\ &= 58 \end{aligned}$$

$$S = 25 \left(\frac{10 + 58}{2} \right) = \boxed{850 \text{ seats}}$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Fill in the blanks to complete the statement.

6) The graph of $y = (x - 10)^2 - 2$ can be obtained from the graph of $y = x^2$ by shifting horizontally ? units to the ? and shifting vertically ? units to the ? direction.

- A) 10; left; 2; upward
C) 2; right; 10; downward

- B) 10; right; 2; downward
D) 10; left; 2; downward

6) B

Determine algebraically whether the function is even, odd, or neither even nor odd.

7) $f(x) = 3x^2 - 3$

A) Neither

B) Even

C) Odd

7) B

even
 $f(x) = f(x)$

odd
 $f(x) = -f(x)$

$f(-x) = 3(-x)^2 - 3 = 3x^2 - 3$

$f(x) = 3x^2 - 3$ even

8) $f(x) = -5x^3 + 8x$

A) Even

B) Odd

C) Neither

8) B

$f(-x) = -5(-x)^3 + 8(-x)$
 $= 5x^3 - 8x$

$-f(x) = -(-5x^3 + 8x)$
 $= 5x^3 - 8x$

odd

Find the zeros of the polynomial function and state the multiplicity of each.

9) $f(x) = 3(x + 6)^2(x - 6)^3$

A) -6, multiplicity 3; 6, multiplicity 2

B) -6, multiplicity 2; 6, multiplicity 3

C) 4, multiplicity 1; -6, multiplicity 3; 6, multiplicity 3

D) 4, multiplicity 1; 6, multiplicity 1; -6, multiplicity 1

$x = -6$, mult. 2

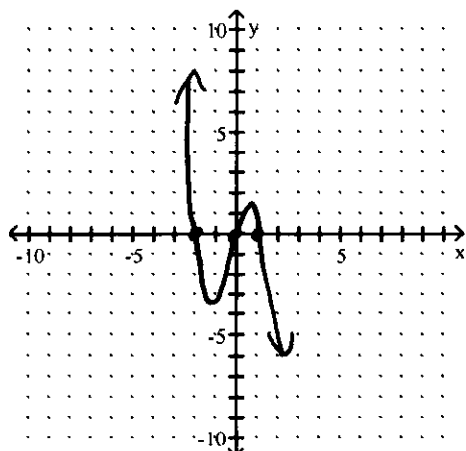
$x = 6$, mult. 3

9) B

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Graph the function.

10) $P(x) = -2x(x - 1)(x + 2)$



Zeros:

$x = 0$, $x = 1$, $x = -2$

End behavior odd + negative
leading coefficient
y-int: (0, 0)

10) _____

Find the remainder when $f(x)$ is divided by $(x - k)$

11) $f(x) = 2x^3 + 3x^2 + 4x + 18; k = -2$

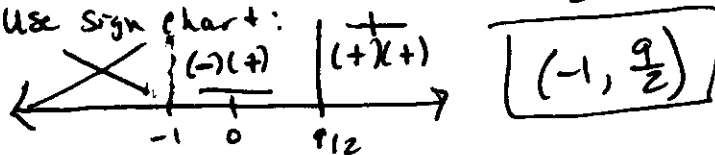
$$\begin{aligned} f(-2) &= 2(-2)^3 + 3(-2)^2 + 4(-2) + 18 \\ &= 2(-8) + 3(4) - 8 + 18 \\ &= -16 + 12 - 8 + 18 = 6 \end{aligned}$$

11) _____

Solve the inequality.

12) $(2x - 9)\sqrt{x + 1} > 0$ *less than zero at $\frac{9}{2}$, $x \geq -1$*

Use sign chart:



12) _____

Find the exact solution to the equation.

13) $\log_4(x - 2) = -1$

$$4^{-1} = x - 2$$

$$\frac{1}{4} = x - 2$$

$$\frac{1}{4} + 2 = x$$

$$x = \frac{9}{4}$$

13) _____

For the given function, find all asymptotes of the type indicated (if there are any)

14) $f(x) = \frac{3}{x^2 - x}$, vertical $\rightarrow x^2 - x \neq 0$
 $x(x - 1) \neq 0$
 $x \neq 0, x \neq 1$

So vertical asymp. at $x = 1$ & $x = 0$

14) _____

Solve the problem.

15) Estimate graphically the local maximum and local minimum of $f(x) = 4x^2 - 2x + 5$.

Graph on calculator and calculate.

Local minimum at 4.75.

No local maximum.

15) _____

State the domain of the rational function.

16) $f(x) = \frac{x - 7}{x^2 + 2x}$

$$x^2 + 2x \neq 0$$

$$x(x + 2) \neq 0$$

$$x \neq 0, x \neq -2$$

$$\text{Domain } (-\infty, -2) \cup (-2, 0) \cup (0, \infty)$$

16) _____

Determine if the function is bounded above, bounded below, bounded on its domain, or unbounded on its domain.

17) $y = -\sin(3x) + 2$

17) _____

graph function to determine.

Bounded on its domain.

Using the given zero, find all other zeros of $f(x)$.

18) i is a zero of $f(x) = x^4 - 4x^3 + 2x^2 - 4x + 1$

18) _____

If $x = i$, then $x = -i$, so $(x-i)(x+i) = x^2 + 1$ is a factor.

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2} = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$$\begin{array}{r} x^2 - 4x + 1 \rightarrow \text{factor, find zeros of this factor using quad. formula.} \\ x^2 + 1 \overline{) x^4 - 4x^3 + 2x^2 - 4x + 1} \\ \underline{-(x^4 + x^2)} \\ -4x^3 + x^2 - 4x + 1 \\ \underline{-(-4x^3 + x^2 - 4x)} \\ x^2 + 1 \end{array}$$

Write a linear factorization of the function.

19) $f(x) = x^3 + 8x^2 + 16x + 128$

19) _____

Find possible zero. use synth. division.

$$\begin{array}{r|rrrr} -8 & 1 & 8 & 16 & 128 \\ & & -8 & 0 & -128 \\ \hline & 1 & 0 & 16 & 0 \end{array} \rightarrow x^2 + 16 = (x+4i)(x-4i)$$

$$f(x) = (x+4i)(x-4i)(x+8)$$

Convert from degrees to radians. Use the value of π found on a calculator and round answers to four decimal places, as needed.

20) 30°

20) _____

$$30^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{\pi}{6}}$$

Convert the radian measure to degree measure. Use the value of π found on a calculator and round answers to two decimal places.

21) $\frac{7\pi}{10}$

21) _____

$$\frac{7\pi}{10} \cdot \frac{180}{\pi} = \boxed{126^\circ}$$

Evaluate without using a calculator.

22) $\sin \theta$, if $\cos \theta = \frac{2}{5}$ and $\tan \theta < 0 \rightarrow$ so, y is negative, if x is positive.

22) _____

$$\sin \theta = \frac{y}{r}$$

$$x = 2$$

$$\cos \theta = \frac{x}{r}$$

$$r = 5$$

$$y = \pm \sqrt{5^2 - 2^2}$$

$$y = \pm \sqrt{21}$$

$$\sin \theta = -\frac{\sqrt{21}}{5}$$

Solve the equation.

23) Solve $\sin \theta = \frac{1}{2}$ for θ , where $0^\circ \leq \theta \leq 90^\circ$.

23) _____

$\theta = 30^\circ$

Suppose that θ is in standard position and the given point is on the terminal side of θ . Give the exact value of the indicated trig function for θ .

24) (9, 12); find $\sin \theta$.

24) _____

$x = 9$
 $y = 12$
 $r = \sqrt{9^2 + 12^2} = 15$
 $\sin \theta = \frac{y}{r} = \frac{12}{15} = \frac{4}{5}$

Use basic identities to simplify the expression.

25) $\cot \theta \sec \theta \sin \theta$

25) _____

$\downarrow \quad \downarrow$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} = 1$

26) $\cos \theta - \cos \theta \sin^2 \theta$

26) _____

Factor: $\cos \theta (1 - \sin^2 \theta) = \cos \theta (\cos^2 \theta) = \cos^3 \theta$

Write each expression in factored form as an algebraic expression of a single trigonometric function.

27) $\csc^2 x - 1 = (\csc x - 1)(\csc x + 1)$

27) _____

Find all solutions in the interval $[0, 2\pi)$.

28) $\cos^2 x + 2 \cos x + 1 = 0$

28) _____

Factor: $(\cos x + 1)^2 = 0$

$\cos x + 1 = 0$

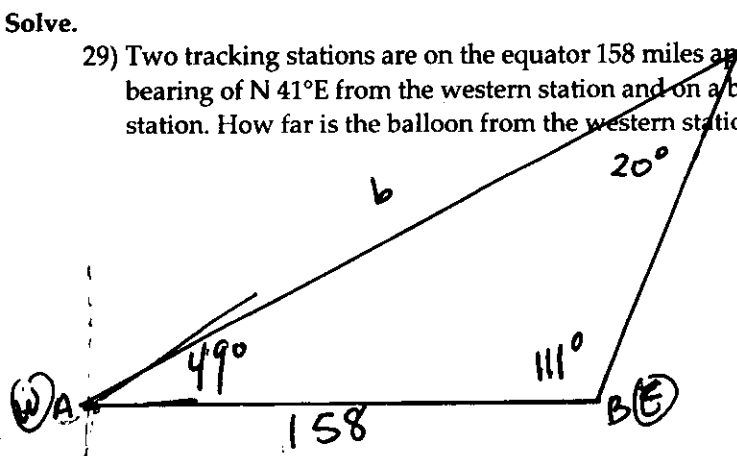
$\cos x = -1$

$x = \pi$

Solve.

29) Two tracking stations are on the equator 158 miles apart. A weather balloon is located on a bearing of N 41° E from the western station and on a bearing of N 21° E from the eastern station. How far is the balloon from the western station?

29) _____



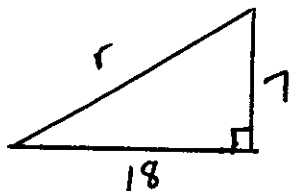
$\frac{\sin 20^\circ}{158} = \frac{\sin 111^\circ}{b}$

$b = \frac{158 \sin 111^\circ}{\sin 20^\circ} \approx 431.3 \text{ miles}$

Solve the problem.

- 30) A building has a ramp to its front doors to accommodate the handicapped. If the distance from the building to the end of the ramp is 18 feet and the height from the ground to the front doors is 7 feet, how long is the ramp? (Round to the nearest tenth.)

30) _____



$$18^2 + 7^2 = r^2$$

$$r = \sqrt{373} \approx \boxed{19.3 \text{ ft.}}$$

Find the vertex, the focus, and the directrix of the parabola.

31) $y^2 - 8x + 8y + 40 = 0$

31) _____

$$\left(\frac{y}{2}\right)^2 = 16y^2 + 8y + \frac{+16}{8x-24} = 8x-40+16$$

$$(y+4)^2 = 8(x-3) \quad a=2$$

vertex: $(3, -4)$ directrix: $x = 1$

focus: $(5, -4)$

Find the center, vertices, and foci of the ellipse with the given equation.

32) $\frac{2x^2}{16} + \frac{8y^2}{16} = 16$

32) _____

center: $(0, 0)$

vertices: $(\sqrt{8}, 0)(-\sqrt{8}, 0) \rightarrow (2\sqrt{2}, 0)(-2\sqrt{2}, 0)$

$$\frac{x^2}{8} + \frac{y^2}{2} = 1$$

foci: $(\sqrt{6}, 0)(-\sqrt{6}, 0)$

$a = \sqrt{8} \quad c^2 = a^2 - b^2$

$b = \sqrt{2} \quad c^2 = 6, c = \sqrt{6}$

33) $\frac{(x+5)^2}{225} + \frac{(y-3)^2}{81} = 1$

33) _____

$a=15, b=9$

center: $(-5, 3) \quad c=12$

vertices: $(10, 3)(-20, 3)$

foci: $(7, 3)(-17, 3)$

Find an equation in standard form for the hyperbola that satisfies the given conditions.

- 34) Vertices at $(0, \pm 3)$, foci at $(0, \pm 9)$

34) _____

center: $(0, 0)$

$a=3 \quad c=9, b^2=72$

$$\frac{y^2}{9} - \frac{x^2}{72} = 1$$

Graph the function without a calculator in order to answer the following questions:

On what intervals is the function increasing? decreasing?

Is the function odd, even, or neither (symmetry)?

Give the function's extrema (maximum and minimum), if any.

Find the horizontal & vertical asymptotes, if any.

Find the domain and range.

Describe end behavior using limits.

Is the function continuous?

Is the function bounded?

35) $f(x) = \frac{x^2}{x^2 - 9}$

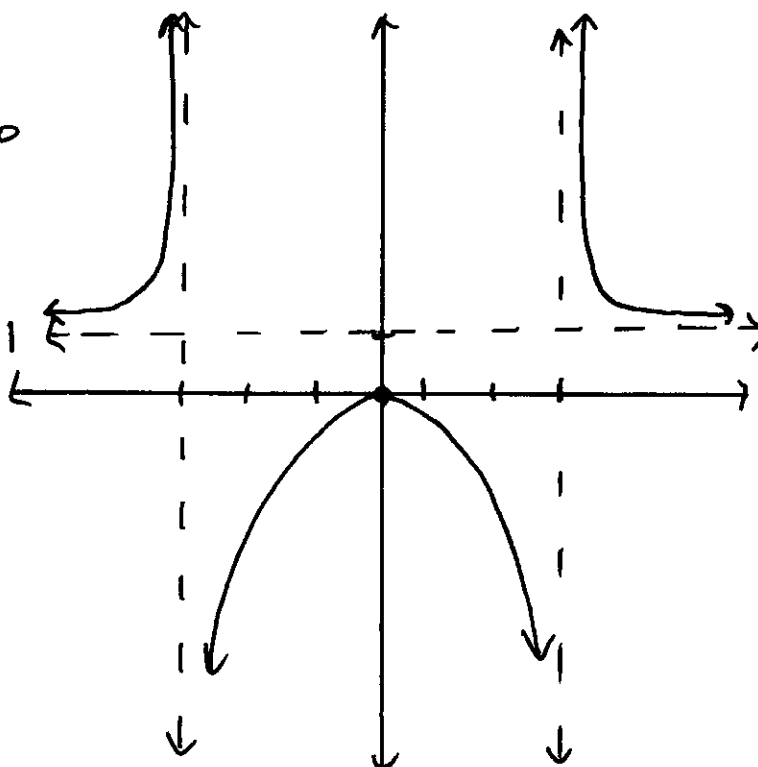
$$x^2 - 9 \neq 0$$

$$(x+3)(x-3) = 0$$

$$x \neq -3, x \neq 3$$

vert. asymptotes
 $x=3, x=-3$

Horiz. asympt. $\frac{x^2}{x^2} \rightarrow 1$
 $y=1$



35) _____

Increasing: $(-\infty, -3) \cup (-3, 0)$

decreasing: $(0, 3) \cup (3, \infty)$

Even symmetry

Local max. at $(0, 0)$

No local min.

vert. asympt. $x=3, x=-3$

Horiz. asympt. $y=1$

Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Range: $(-\infty, 0] \cup (1, \infty)$

End Behavior:

$$\lim_{x \rightarrow \infty} f(x) = 1 \quad \lim_{x \rightarrow -\infty} f(x) = 1$$

Not continuous

Not Bounded